PROPOSED TITLE OF THE RESEARCH

STUDY ON ENTIRE AND MEROMORPHIC FUNCTIONS

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SYNOPSIS

Keywords: Entire functions, Meromorphic functions, Order, Relative order, Proximate order, Growth.

For two transcendental entire functions f and g defined in the open complex plane $\mathbb{C}$, Clunie [2] proved that $\lim_{r\to\infty}\frac{T(r,fog)}{T(r,f)}=\infty$ and $\lim_{r\to\infty}\frac{T(r,fog)}{T(r,g)}=\infty$. After this in [6], Singh investigated some comparative growth properties of $\log T(r,fog)$ and $T(r,f)$ and raised the question for comparative growth of $\log T(r,fog)$ and $T(r,g)$. However, some results on comparative growth of $\log T(r,fog)$ and $T(r,g)$ are closely investigated in [3] and also in [1].
Let $f(z)$ be a non-constant meromorphic function in the finite complex plane and $a_0(z),a_1(z),...,a_l(z)$ are all small functions of $f(z)$ i.e., $T(r,a_i(z))=o\{T(r,f)\};i=0,1,...,l$. We form the meromorphic function $\Psi(z)$ defined as
$\Psi(z)=\Sigma_{i=0}^{l}a_i(z)f^{(i)}(z)$, where $f^{(i)}$ is the i-th derivative of $f(z)$.
If f is a meromorphic function then the order ρ_f and lower order λ_f of f are defined as $\rho_f=\limsup_{r\to\infty}\frac{logT(r,f)}{logr}$ and $\lambda_f=\liminf_{r\to\infty}\frac{logT(r,f)}{logr}$, where $T(r,f)$ is the Nevanlinna characteristic function of $f(z)$.
In [5], Sato hinted the definition of generalised order of f as

$$\rho_k=\limsup_{r\to\infty}\frac{log^{[k-1]}T(r,f)}{logr}.$$

Also the hyper order $\overline{\rho_f}$ and hyper lower order $\overline{\lambda_f}$ of a meromorphic function f are defined as $\overline{\rho_f}=\limsup_{r\to\infty}\frac{log^{[2]}T(r,f)}{logr}$ and $\overline{\lambda_f}=\liminf_{r\to\infty}\frac{log^{[2]}T(r,f)}{logr}$.
The definition of hyper order and hyper lower order $\overline{\rho_f}$ and $\overline{\lambda_f}$ are meaningful only if the order $\rho_f=\infty$.

With these definitions of order and hyper order we have proved the following results:
Let $f(z)$ be a non-constant meromorphic function and $g(z)$ be an entire function. Then

$$\liminf_{r\to\infty}\frac{logT(r,\Psi og)}{T(r,g)}\leq 3\rho_f 2^{\lambda_g}.$$

Further if there exist entire functions $b_i(i=1,2\ldots,n;n\leq\infty)$ such that $T(r,b_i)=o\{T(r,g)\}$ with $\Sigma_{i=1}^{n}\delta(b_i,g)=1$, then

$$\limsup_{r\to\infty}\frac{logT(r,\Psi og)}{T(r,g)}\leq\pi\rho_f.$$

Also if $\Psi(z)$ is of positive lower order and $0<\lambda_g\leq\rho_g<\infty$, then

$$(i)\liminf_{r\to\infty}\frac{log^{[2]}T(r,\Psi og)}{T(exp(r)^{\mu},g)}\leq\frac{\overline{\lambda_g}}{\rho_g},\text{ when }0<\lambda_g<\mu<\infty;$$
$$\text{and}\quad (ii)\liminf_{r\to\infty}\frac{log^{[2]}T(r,\Psi og)}{T(exp(r)^{\mu},g)}\geq\frac{\overline{\rho_g}}{\lambda_g},\text{ when }0<\mu<\rho_g<\infty.$$

If g be an irregular growth function i.e., $\lambda_g < \mu < \rho_g$, then

$$\liminf_{r\to\infty} \frac{log^{[2]}T(r,\Psi og)}{logT(exp(r)^{\mu},g)} \leq \frac{\overline{\lambda_g}}{\rho_g} \leq \frac{\overline{\rho_g}}{\lambda_g} \leq \limsup_{r\to\infty} \frac{log^{[2]}T(r,\Psi og)}{logT(exp(r)^{\mu},g)}.$$

Under the same hypothesis we have proved

$$\text{(i) } \liminf_{r\to\infty} \frac{logT(r,\Psi og)}{logT(exp(r)^{\mu},g)} \leq \frac{\lambda_g}{\rho_g}, \text{ when } 0 < \lambda_g < \mu < \infty;$$
$$\text{and} \quad \text{(ii) } \liminf_{r\to\infty} \frac{logT(r,\Psi og)}{logT(exp(r)^{\mu},g)} \geq \frac{\rho_g}{\lambda_g}, \text{ when } 0 < \mu < \rho_g < \infty.$$

If g be an irregular growth function i.e., $\lambda_g < \mu < \rho_g$, then

$$\liminf_{r\to\infty} \frac{logT(r,\Psi og)}{logT(exp(r)^{\mu},g)} \leq \frac{\lambda_g}{\rho_g} \leq \frac{\rho_g}{\lambda_g} \leq \limsup_{r\to\infty} \frac{logT(r,\Psi og)}{logT(exp(r)^{\mu},g)}.$$

Further we have proved that if $f(z)$ be a non-constant meromorphic function of finite order in the finite complex plane and $g(z)$ be an entire function satisfying $0 < \lambda_g \leq \rho_g < \infty$, then

$$\lim_{r\to\infty} \frac{logT(r,\Psi og)}{T(expr,g^{(k)})} = 0,$$

for $k = 0, 1, 2, \ldots$.
We also see that the condition $\rho_f < \infty$ is necessary for the above result.

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